with Chu's eigenvalues (which as we noted proved remarkably accurate for the case of $e = 0.75$), but rather that the correct field pattern for the $TM_{01}$ mode was *qualitatively* radically different from Chu's; neither Rembold's paper, nor any of those referred to by Dr. Davies, addresses the question of the field shape of the $TM_{01}$ mode. We should also note that, having found Chu's figure reproduced in the 1986 reprinting of Marcuvitz's book (which, we are informed by colleagues, is something of a "bible" in the field), with no mention of any associated erratum, we restricted our literature search to papers published after 1985, and, in any case, to papers showing the actual field shapes (we found none). Although it was not the purpose of our note to present a comprehensive study of the elliptical waveguide, we nonetheless welcome Dr. Davies' suggestions for remedying the noted deficiencies in our bibliography.

Finally a note on the "demystifying" of Mathieu functions. It has been our experience that not all of our colleagues are as conversant with these functions as Dr. Davies obviously is, and our remark was intended as a somewhat light-handed way of acknowledging this fact. We regret any offense that may have been given; none was intended.

Many of the above remarks are equally applicable to the comments of Wiltse and Gfroerer. In particular, [11] in their article, which is said to summarize the various corrections to the Chu paper, refers exclusively to wave-impedance calculations and makes no reference whatsoever to field shapes.

The work described in [6] in their article (the 1971 Kretzschmar paper) is another matter. As we stated in replying to Dr. Davies's comments, we relied on what a number of electrical engineering colleagues advised us was the standard reference work (Marcuvitz's *Waveguide Handbook*,) and only searched the *subsequent* literature, so we were indeed ignorant of Kretszchmar's work on the error in the field shapes. While it does not fully exonerate us, we find our ignorance of this subsequent work places us in rather learned company: In addition to the three referees of our paper, we would add Dr. Julius Stratton (Chu's thesis advisor), Dr. N. Marcuvitz, and, apparently, Kretzschmar's co-author on a 1972 paper on elliptical waveguides, one J. B. Davies. (Whether we should be similarly faulted for our ignorance of an unpublished 1962 thesis from the Aachen Technische Hochschule, we leave to the judgment of your readers.)

Part of the difficulty seems to be the lack of "standard references" which are up to date; despite the apparent "textbook" nature of the elliptical waveguide problem, none of the sources for the corrections referred to by either Davies or Wiltse and Gfroerer is such a source. In fact one of our main motivations in writing the paper was to point out a qualitative error that had persisted in the latest edition of one of the most heavily relied on standard sources. Indeed, our decision to publish in IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, rather than the *Journal of Applied Physics* (in which Chu's paper originally appeared), was to bring the correction to the attention of the widest possible audience. We feel fairly certain that the combination of our article and the lively correspondence it has generated will achieve that goal, if not precisely in the way originally intended.

In closing, we would like to thank the authors of the two letters for their interest and comments, and, since we have not yet explicitly done so, to extend our apologies to Dr. Kretzschmar for inadvertently taking the credit which is rightfully his.

## Comments on "Full-Wave Analysis of Discontinuities in Planar Waveguides by the Method of Lines Using a Source Approach"

Ling Chen

I think there are some mistakes in the above paper[1]. The method presented in that article is unavailable. Because (2) in the article should be

$$\psi_0^{e+} = -\psi_0^{e-} = \frac{\psi_0^e}{1 - r} \quad \text{and} \quad \psi_0^{h+} = \psi_0^{h-} = \frac{\psi_0^h}{1 + r}. \tag{2}$$

According to this, (3) should be

$$\left.\frac{\partial \psi^h}{\partial z}\right|_{z=0} = -j\beta \frac{1 - r}{1 + r} \psi_0^h. \tag{3}$$

Equation (15) should be

$$\left[h^2 \frac{\partial^2 \psi^{h2}}{\partial z^2}\right] r_{zh}^{-1} = \left[h \frac{\partial \psi^h}{\partial z}\right] r_{ze}^{-1} D_z^t - \left.\left[h \frac{\partial \psi^h}{\partial z}\right]\right|_{sz=0} r_{zh1}$$

$$= -\psi^h r_{zh}^{-1} D_z D_z^t + j\beta \frac{1 - r}{1 + r} h\psi_0^h r_{zh1}. \tag{15}$$

Equation (16) should be

$$r_{xh}^{-1}\left[h^2 \frac{\partial^2 \psi^h}{\partial z^2}\right] r_{xh}^{-1} \rightarrow -\phi^h D_z D_z^t + j\beta \frac{1 - r}{1 + r} h\phi_0^h$$

$$= \phi^h D_{zz}^{ht} + j\beta \frac{1 - r}{1 + r} h\phi_0^h. \tag{16}$$

Equation (32) should be

$$\frac{d^2 \phi^h}{dy^2} + \frac{D_{xx}^h \phi^h}{h^2} + \frac{\phi^h D_{zz}^{ht}}{h^2} + \epsilon_r k_0^2 \phi^h = \frac{-j\beta h \phi_0^h}{h^2} \frac{1 - r}{1 + r}. \tag{32}$$

Equation (33) should be

$$[V_p]_{ik} = \frac{-j\beta h [V_0]_{ik}}{([k_0]_i^2 - k_{ik}^2) h^2} \frac{1 - r}{1 + r}. \tag{33}$$

Equation (34) should be

$$h \left.\frac{d[V]_{ik}}{dy}\right|_{y=0} = -[\gamma^h]_{ik} [V]_{ik} + j\beta \frac{1 - r}{1 + r} h[\gamma_q^h]_{ik} [V_0]_{ik}. \tag{34}$$

Therefore, $\tilde{Z}_q$ in (37) and $Z_{q,\text{red}}$ in (38) are related to the unsolved parameter $r$. So, the current distribution cannot be obtained from (38), the reflection coefficient and the normalized input impedance cannot be obtained.

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[1] S. B. Worm, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1510–1514, Oct. 1990.

## Author's Reply[2]

Stephan B. Worm

Ling Chen does not give any arguments why (2) should be like he suggests, but maybe the following explanation can make my definition more clear.

If we write the solution for a wave propagating in positive $z$-direction as

$$\begin{pmatrix} \psi_o^e \\ \psi_o^h \end{pmatrix}$$

then a solution for a wave in negative $z$-direction can be written as

$$\begin{pmatrix} -\psi_o^e \\ \psi_o^h \end{pmatrix}.$$

A general wave is now represented by

$$\begin{pmatrix} \psi^e \\ \psi^h \end{pmatrix} = A \begin{pmatrix} \psi_o^e \\ \psi_o^h \end{pmatrix} e^{-j\beta z} + B \begin{pmatrix} -\psi_o^e \\ \psi_o^h \end{pmatrix} e^{j\beta z}.$$

The potential function $\psi^h$ has a $z$-dependency like $E_y$ and thus like the voltage between a microstrip and ground. For the voltage reflection coefficient at $z = 0$ we obtain $r = B/A$.

With the discretization method we can impose a Dirichlet boundary condition for $\psi^e$ and a Neumann condition for $\psi^h$ (or reverse, but not twice the same condition):

$$\psi^e(z = 0) = A\psi_o^e + rA(-\psi_o^e) = (1 - r)A\psi_o^e.$$

With a normalization to $A = 1/(1 - r)$ we can use the solution $\psi_o^e$ as obtained from the propagation problem at the boundary $z = 0$.

It then follows that

$$\partial\psi^h/\partial z \; (z = 0) = -j\beta\psi_o^h.$$

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## Comments on "An Analytic Algorithm for Unbalanced Stripline Impedance"

Robert E. Canright, Jr.

*Abstract*—**This letter corroborates the results of recent research, promotes an alternative technique for calculating the impedance of unbalanced stripline, and highlights some older references that are often overlooked.**

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IEEE Log Number 9103910.
[1]P. Robrish, *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 8, pp. 1011-1016, Aug. 1990.

This author commends P. Robrish for solving the conformal mapping problem for the unbalanced stripline in the above paper.[1] Finding the easiest technique to calculate the characteristic impedance of unbalanced stripline is an important practical problem that has been under assault for some time [1]. Robrish's work fills an important hole in the theory. For the sake of completeness, someone needed to work through the conformal mapping, extending Cohn's [2] earlier work, and give us a solution for a stripline that is not centered between the ground planes. However, this author recommends a different technique that produces equivalent results, is easier to use, and has a flexibility that makes it more powerful [3]. First, the alternative will be presented, then Robrish's algorithm will be corroborated, then this letter will conclude with some discussion.

The alternate approach is very easy to state:

$$Z_0 = 2\, Z_{01} Z_{02}/(Z_{01} + Z_{02}) \tag{1}$$

where

$Z_{01}$ = the stripline impedance based on the distance to the near ground plane

$Z_{02}$ = the stripline impedance based on the distance to the far ground plane

as shown in Fig. 1.

Looking at Fig. 1, the reader should notice that two line widths can be accommodated. This means that the sloping side walls that sometimes appear in printed wiring board (PWB) conductors can be accounted for when calculating the line impedance. Gupta [1] had looked for an easy algorithm to account for this effect, which occurs when the copper conductor is etched and is commonly called "undercut." Fig. 2 shows that conventional stripline impedance can also account for undercut effects by using this alternate technique. Undercut is an effect unaccounted for in Robrish's technique, which is why this author suggests that this new technique is more powerful. The phrase "new technique" means that the reader is expected to be unfamiliar with it, not that it is recent. This alternate technique was first presented without proof in 1987 [4], with its derivation [3] published later (April 1990).

Table I shows the comparison between Robrish's formulas and the use of (1) when analyzing six designs. Robrish's formulas were used to create the six designs, which is why the impedances have such tidy values when analyzed with the same formula used to create the design. To emphasize how the accuracy of (1) depends upon the stripline formulas used to calculate $Z_{01}$ and $Z_{02}$, both Wheeler's technique [5] and Cohn's technique [2] were used for the analysis. The differences in impedance between Robrish's formulas and (1) are generally less than or equal to 1 percent, certainly within the 2 percent accuracy of Robrish's formulas as cited in the conclusion of his very fine paper. Hence, the results of (1), in conjunction with Cohn's technique, are essentially equivalent to Robrish's formulas.

Regarding Table I: $B$ is the distance in mils between ground planes. The distance between the conductor and the near ground plane is $B/3$ ($cl = b/3$ using Robrish's terms). The conductor width, $w$, is 5.00 mil. The conductor thickness, $t$, is 1.4 mil. The PWB dielectric constant is 4.8. The impedance is in ohms.

Cohn's and Robrish's techniques require a round wire approximation to the rectangular conductor. Robrish uses

$$D = (2/3)(0.8w + t). \tag{2}$$